The Steady-State Structure of the Ultralong Waves Produced by Heating With A Pressure-Dependent Frictional Effect

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ABSTRACT—A simple linear model was used to determine the influence of a pressure-dependent friction term on the steady-state solution for long waves produced by heating. Resonance effects, which are found in a frictionless atmosphere, were also evident in the case when friction was concentrated in the boundary layer. As friction increased at low pressure, resonance effects were no longer apparent.

Net potential and kinetic energy changes did not occur when friction was zero. When friction was allowed, the conversion term for potential energy was balanced by the generation term and the northward heat transport term. The net frictional loss of kinetic energy was balanced by the conversion of potential energy into kinetic energy.

1. INTRODUCTION

This study presents further research on the behavior of long waves in the atmosphere under the influence of heating. Aspects of this subject have received considerable attention in the literature (Charney and Eliassen 1949, Smagorinsky 1953, Saltzman 1965, Murakami 1967, Derome and Wiin-Nielsen 1971, and Chow et al. 1972). The primary purpose of the study is to examine the structure of the steady-state solution when a pressure dependent friction term is included.

In an earlier study (Chow et al. 1972), a frictionless atmosphere was considered, and the behavior of the pressure integrated disturbances in the β-plane were studied using linear theory. This research, also using linear theory and β-plane representation, considers the internal structure of the atmosphere instead of integrated effects.

2. BASIC FORMULATION

The structure of the atmosphere was derived through the steady-state vorticity equation and thermodynamic equation; that is,

$$\mathbf{V}_H \cdot \nabla_H \xi + \omega \frac{\partial \xi}{\partial p} + \beta v = (\xi + f) \frac{\partial \omega}{\partial p} - K\xi \tag{1}$$

and

$$\sigma\omega = \mathbf{V}_H \cdot \nabla \frac{\partial \phi}{\partial p} + \frac{R}{c_{pd}} \frac{Q}{p}. \tag{2}$$

The last term on the right of eq (1) is the friction term. K is the coefficient of friction, which was assumed to vary exponentially with pressure; that is, $K = K_0 \exp[-m(p_0 - p)/p_0]$, where $p_0 = 100$ cb, K_0 is the value of K at $p = p_0$, and m is the rate constant. The term ξ is the vertical component of the relative vorticity, σ is the mean stability term, Q is the rate at which heat is added per unit mass, and $\mathbf{V}_H$ is the horizontal component of the wind velocity. The rest of the terms in the equations have the standard meanings assigned to them in meteorology.

Perturbation principles and the condition that variations of the perturbation in the y direction are zero were applied to eq (1) and (2). The resulting equations were linearized. It was assumed that the horizontal component of the wind, v', could be approximated by a stream function, ψ (i.e., $v' = \partial\psi/\partial x$), and that $\phi \approx f\psi$. The vorticity and thermodynamic equations were then

$$\overline{U} \frac{\partial^3 \psi}{\partial x^3} + \beta \frac{\partial \psi}{\partial x} = f \frac{\partial \omega'}{\partial p} - K \frac{\partial^2 \psi}{\partial x^2} \tag{3}$$

and

$$\sigma\omega' = f\overline{U} \frac{\partial^2 \psi}{\partial p \partial x} - \frac{f\overline{U}_{T4}}{\Delta p} \frac{\partial \psi}{\partial x} + \frac{R}{c_{pd}} \frac{Q'}{p}. \tag{4}$$

Here, the mean zonal wind, $\overline{U}$, was assumed to vary linearly with pressure; that is, $\overline{U} = \overline{U}_4 + \overline{U}_{T4}\,(p - p_4)/\Delta p$, where $\overline{U}_4$ is the wind speed at $p = p_4 = 60$ cb, $\overline{U}_{T4}$ is the thermal wind at p_4, and $\Delta p = -80$ cb.

Assuming $Q' = H_0 f(p)$, where H_0 is the surface heating and $f(p)$ is some function of pressure, and eliminating ω' from eq (3) using eq (4), yields

$$\overline{U} \frac{\partial^3 \psi}{\partial x^3} + \beta \frac{\partial \psi}{\partial x} + \overline{U} \Delta p \mu_1 \frac{\partial^3 \psi}{\partial p^2 \partial x} + K \frac{\partial^2 \psi}{\partial x^2} = \mu_2 H_0 F(p) \tag{5}$$

where

$$\mu_1 = -\frac{f^2}{\Delta p \sigma},$$

$$\mu_2 = -\frac{\mu_1 \Delta p R}{f c_{pd}},$$

and

$$F(p) = \frac{\partial}{\partial p}\left[\frac{f(p)}{p}\right].$$

The following solutions were assumed:

$$\psi=\sum_{1}^{\infty}\psi_1 \cos kx+\psi_2 \sin kx \tag{6}$$

and

$$H_0=\sum_{1}^{\infty}h_1 \cos kx+h_2 \sin kx \tag{7}$$

where ψ_1 and ψ_2 are Fourier coefficients for the stream function; that is,

$$\psi_1=\bar{\psi} \cos \alpha,$$

$$\psi_2=\bar{\psi} \sin \alpha,$$

and

$$\bar{\psi}=\sqrt{\psi_1^2+\psi_2^2}, \tag{8}$$

and α is the phase angle for the stream function (i.e., $\alpha=\arctan \psi_2/\psi_1$). For the surface heating, h_1 and h_2 are also Fourier coefficients; that is,

$$h_1=h \cos \alpha_H,$$

$$h_2=h \sin \alpha_H,$$

and

$$h=\sqrt{h_1^2+h_2^2}, \tag{9}$$

and α_H is the phase angle for the surface heating wave (i.e., $\alpha_H=\arctan h_2/h_1$). The term k is the wave number (i.e., $k=2\pi n/L$), L is the total wavelength, and n is the nondimensional wave number ($n=1, 2, 3, \ldots$). Substitution of eq (6) and (7) into eq (5) yields

$$\frac{\partial^2\psi_1}{\partial p^2}-\frac{\left(\bar{U}-\frac{\beta}{k^2}\right)k^2}{\bar{U}\Delta p\mu_1}\psi_1=\frac{\mu_2}{\mu_1}\frac{h_2}{\bar{U}\Delta pk}F(p)-\frac{Kk}{\bar{U}\Delta p\mu_1}\psi_2 \tag{10}$$

and

$$\frac{\partial^2\psi_2}{\partial p^2}-\frac{\left(\bar{U}-\frac{\beta}{k^2}\right)k^2}{\bar{U}\Delta p\mu_1}\psi_2=\frac{-\mu_2}{\mu_1}\frac{h_1}{\bar{U}\Delta pk}F(p)+\frac{Kk}{\bar{U}\Delta p\mu_1}\psi_1 \tag{11}$$

When $K=0$ and the coefficients of the above equation are independent of pressure, it can be shown that the solution will approach infinity when $\bar{U}-\beta/k^2$ approaches zero. This is the state of resonance. Figure 1 yields the wave number at which resonance can be found for a given mean wind speed at 45° latitude.

A nine-level model was used to solve the above equations. The lower boundary was $p=p_0=100$ cb, the upper boundary was $p=p_8=20$ cb, and the pressure levels were spaced at 10-cb increments. Many different values of $\bar{U}_4$ and $\bar{U}_{T4}$ were used to solve the equations; however, only the solutions for $\bar{U}=10$ m/s and $\bar{U}_{T4}=20$ m/s are presented. Changing $\bar{U}_4$ and $\bar{U}_{T4}$ did not change the general characteristics of the solutions, which are discussed in the following sections.

Solutions were obtained for the following three states of frictional influence: (1) $K_0=0$, which is the state of no friction; (2) $m=15$ and $K_0=10^{-4}\text{s}^{-1}$, which is a state wherein only the boundary layer has appreciable friction;

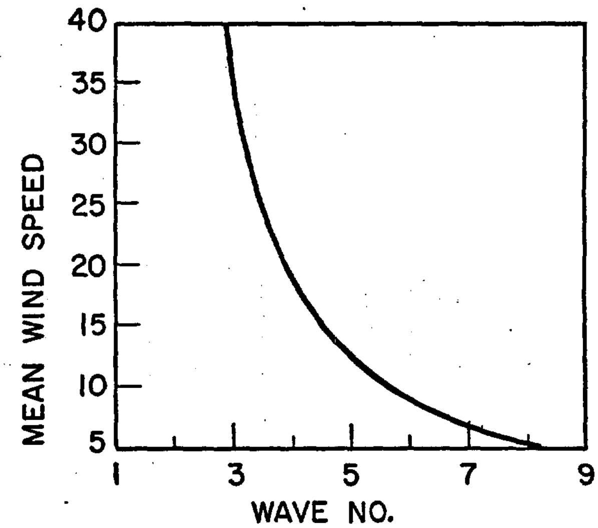

FIGURE 1.—Wave number for nonfrictional resonance versus mean wind speed (m/s).

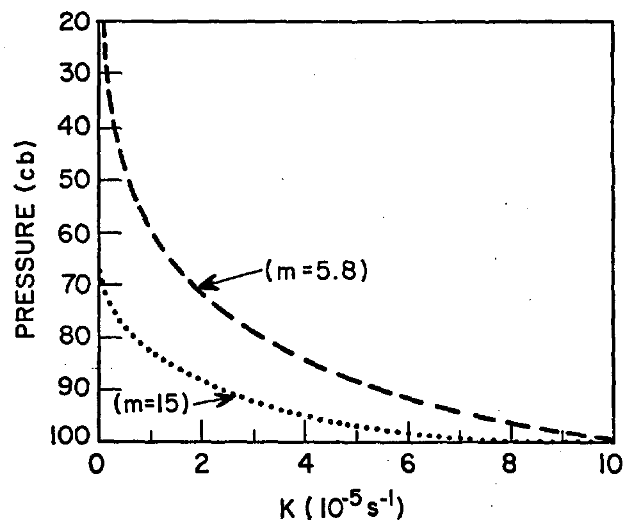

FIGURE 2.—Distribution of the friction coefficient with pressure for $m=15$ and $m=5.8$.

and (3) $m=5.8$ and $K_0=10^{-4}\text{s}^{-1}$, which is a state wherein there is significant friction in the upper atmosphere (fig. 2). In all cases, a standard atmospheric lapse rate was used in the computation of the mean stability term, σ. The latitude was 45°.

Heating was assumed to vary linearly with pressure, and to be zero at $p=p_8$; that is, $f(p)=(p_8-p)/\Delta p$ and $F(p)=(-p_8/\Delta p\ p^2)$. The magnitude of the surface heating rate for each wave number was assumed to be 0.01 kJ $\cdot t^{-1}\cdot s^{-1}$, which is about one-tenth the value of the surface heating reported by Döös (1961) and Ninomiya (1964). Their heating computations reflected the sum of all harmonics. It was assumed that a value of one-tenth would be characteristic of a single harmonic. The phase angle of the surface heating, α_H, was fixed at zero degrees ($\alpha_H=0$). Application of these conditions to eq (10) and (11) yielded

$$\frac{\partial^2\psi_1}{\partial p^2}-\frac{C_Rk^2}{\bar{U}\Delta p\mu_1}\psi_1=\frac{-Kk}{\bar{U}\Delta p\mu_1}\psi_2 \tag{12}$$

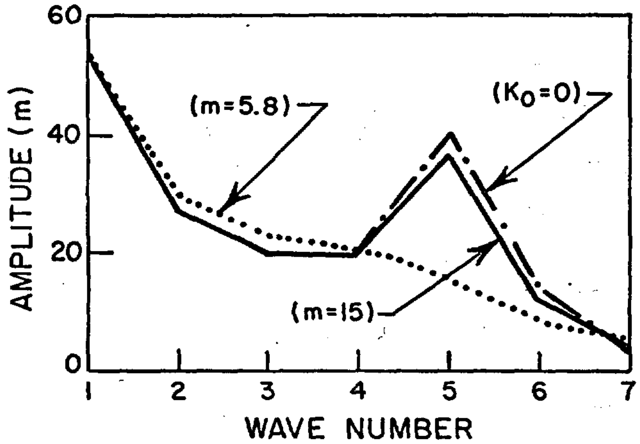

FIGURE 3.—The pressure wave amplitude (m) at 20 cb versus wave number and friction distribution.

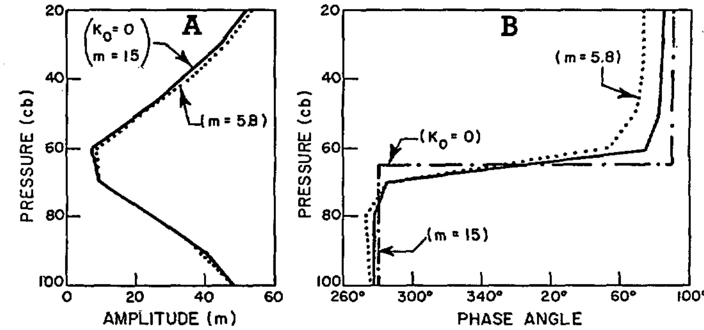

FIGURE 4.—(A) the pressure wave amplitude and (B) the pressure wave phase angle versus pressure for wave number 1.

and

$$\frac{\partial^2\psi_2}{\partial p^2}-\frac{C_R k^2}{\overline{U}\Delta p\mu_1}\psi_2=\frac{\mu_1}{\mu_2}\frac{p_8 h_1}{\overline{U}\Delta p^2 k}\left(\frac{1}{p^2}\right)+\frac{Kk}{\overline{U}\Delta p\mu_1}\psi_1 \tag{13}$$

where $C_R=U-\beta/k^2$ and is the Rossby speed.

The above equations were solved under the following boundary condition. The vertical velocity, ω, was assumed zero ($\omega=0$) at $p=p_0$ and at $p=p_8$. This condition yielded

$$\left.\begin{aligned}\frac{\partial\psi_1}{\partial p}-\frac{\overline{U}_T}{U_0\Delta p}\psi_1&=0\\ \frac{\partial\psi_2}{\partial p}-\frac{\overline{U}_T}{U_0\Delta p}\psi_2&=-\frac{\mu_3 h_1}{\overline{U}_0 k}\end{aligned}\right\}p=p_0$$

and

$$\left.\begin{aligned}\frac{\partial\psi_1}{\partial p}-\frac{\overline{U}_T}{U_8\Delta p}\psi_1&=0\\ \frac{\partial\psi_2}{\partial p}-\frac{\overline{U}_T}{U_8\Delta p}\psi_2&=0\end{aligned}\right\}p=p_8 \tag{14}$$

where $\mu_3=R/c_{pd}fp_0$, and U_0 is the zonal wind speed at p_0. The upper boundary condition assumes that the stable stratosphere dampens all vertical motion. Under these conditions, downward vertical motions were found over the heat source.

3. PRESSURE PERTURBATIONS

Figure 3 shows the amplitude of the height perturbation at $p=p_8$ versus wave number obtained under the conditions previously specified. In the case where $K_0=0$ (no friction), the largest amplitude is found at $n=1$; but a secondary maximum appears at $n=5$, which is near the wavelength of nonfrictional resonance (fig. 1). When friction is introduced and is concentrated in the boundary layer ($m=15$), the influence of resonance is still apparent around wave number 5, but the amplitude is reduced. When considerable friction is maintained in the upper atmosphere ($m=5.8$), the influence of resonance appears to have been eliminated.

Figures 4–6 yield the variation of the amplitude and phase of the height perturbation with pressure for wave numbers 1, 3, and 5, which represent the structure of the ultralong waves. With the exception of wave number 5, which is influenced by resonance, the amplitude at high pressure is reduced when marked friction ($m=5.8$) is introduced and increased at low pressure. This is consistent with the results of Smagorinsky (1953) and Murakami (1967).

In an atmosphere characterized by $\omega=0$ at the upper and lower boundary and net downward motion over the heat source, the internal structure of the atmosphere can be generally described by low-level divergence and upper-level convergence. For the no-friction case, the dynamic transport of momentum is the only term that balances the divergence; that is,

$$-k^2 C_R v'=f\frac{\partial\omega}{\partial p}, \tag{15}$$

which is another form of eq (3) with $K_0=0$. The term on the left is the dynamic transport term, and that on the right is the divergence term. The equation indicates that the magnitude of the height perturbation is governed by divergence alone. When friction is introduced,

$$-k^2 C_R v'=f\frac{\partial\omega}{\partial p}-K\frac{\partial v'}{\partial x}. \tag{16}$$

The frictional term acts opposite to the divergence term at low levels, and the pressure perturbation required to balance this combined effect will be less than that for the no-friction case. In the upper levels, the friction term acts in the same sense as the divergence, and the height perturbation must be larger.

The minimum value of the height perturbation amplitude is found at the level of nondivergence for the no-friction case. When friction is included, the minimum value is found at the level where the divergence balances the frictional effect, and the pressure ridge is found directly over the heat source. The level of nondivergence, where the dynamic transport of momentum is balanced by friction alone, is found at a slightly lower pressure.

The phase angles indicate that the ridge is found upstream of the heat source (divergence zone) at low levels and downstream (convergence zone) at upper levels

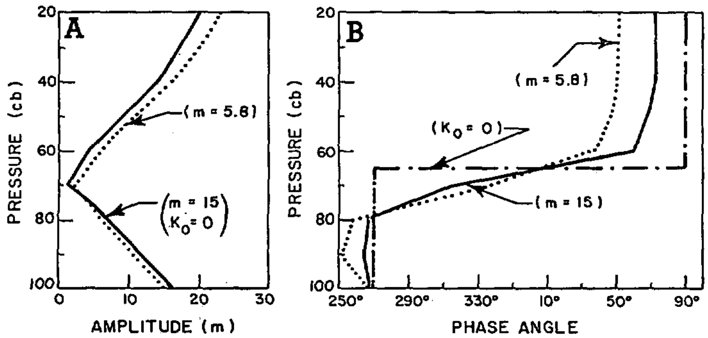

FIGURE 5.—Same as figure 4 for wave number 3.

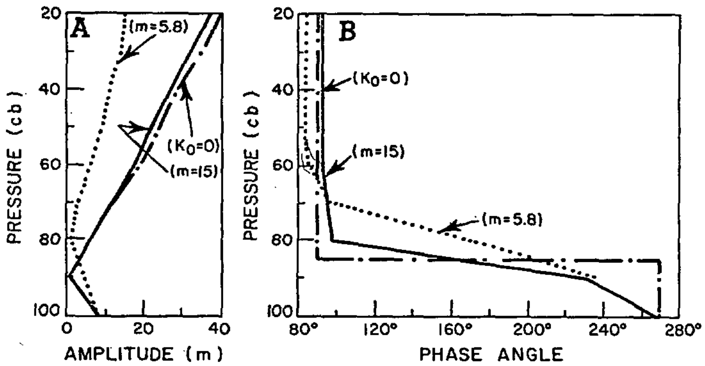

FIGURE 6.—Same as figure 4 for wave number 5.

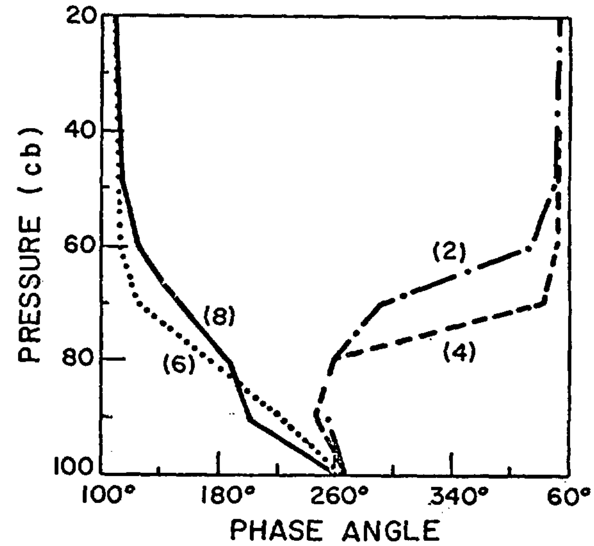

FIGURE 7.—The pressure wave phase angle versus pressure for wave numbers 2, 4, 6, and 8 for $m=5.8$.

for the no-friction case. The change from upstream to downstream location occurs discontinuously at the level of nondivergence where the momentum transport must change signs to balance the divergence. Except near the resonance wave number, the level of nondivergence was located at 75 cb.

When friction was introduced, the transition from upstream to downstream position occurred in a continuous manner (Smagorinsky 1953, Saltzman 1965). For the ultralong waves, there was an eastward tilt of the pressure systems. As friction is increased at low pressure ($m=5.8$), the slope of the pressure systems decreased.

Figure 7 gives the distribution of the phase angle with pressure for $n=2$, 4, 6, and 8 and for $m=5.8$. The shorter waves are included to indicate the transition from an eastward tilt to a westward tilt of the pressure systems. The westward tilt, which first occurs at $n=5$ (fig. 6), is brought about by the change in sign of the pressure-average Rossby speed, which is less than zero ($C_R<0$) for $n<5$ and greater than zero for $n\geq 5$. The changes in sign of the Rossby speed changes the nature of the dynamic transport term requiring a westward displacement of the pressure systems with decreasing pressure to maintain a balanced state.

When horizontal variations are constrained to be unidimensional, C_R will change sign and eastward and westward tilt will occur under the given initial conditions. However, if the horizontal variations are two-dimensional, then the effect of the rotation of the earth can be diminished significantly depending on the wave number and the y scale length, and C_R may be greater than zero for most wave numbers. The ridges and troughs will only slope westward in that case. This is consistent with the results of Smagorinsky (1953), Saltzman (1965), and Derome and Wiin-Nielsen (1971), who found westward slopes in the case of two-dimensional, horizontal variations for all wave numbers.

4. ENERGETICS

Discussions of the effect of friction on the steady-state structure of the atmosphere produced by surface heating would not be complete without describing the energetics. The nature of the changes in potential energy can be studied through eq (4). Applying eq (7) and

$$\omega'=\sum_1^\infty \omega_1 \cos kx+\omega_2 \sin kx,$$

$$\frac{\partial\psi}{\partial p}=\sum^\infty \lambda_1 \cos kx+\lambda_2 \sin kx,$$

$$\lambda_0=\sqrt{\lambda_1^2+\lambda_2^2},$$

$$\omega_0=\sqrt{\omega_1^2+\omega_2^2}$$

$$\alpha_\omega=\arctan\left(\frac{\omega_2}{\omega_1}\right),$$

$$\alpha_p=\arctan\left(\frac{\lambda_2}{\lambda_1}\right),$$

and

$$k\bar{\psi}=\bar{v},$$

to eq (4) and multiplying the equation involving ω_1 by λ_1 and that involving ω_2 by λ_2 yields

$$\sigma\omega_0\lambda_0 \cos(\alpha_\omega-\alpha_p)=\frac{R}{c_{pd}\,p}h\lambda_0\cos(\alpha_H-\alpha_p)$$
$$-\frac{f\overline{U}_{T4}}{\Delta p}k\lambda_0\bar{\psi}\sin(\alpha-\alpha_p). \quad (17)$$

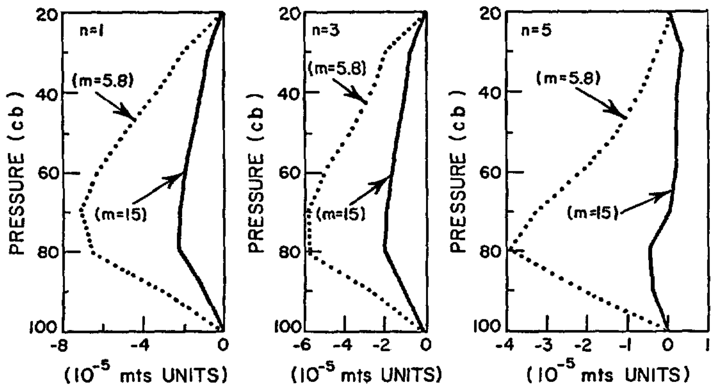

FIGURE 8.—The conversion term for potential energy versus pressure for wave numbers 1, 3, and 5.

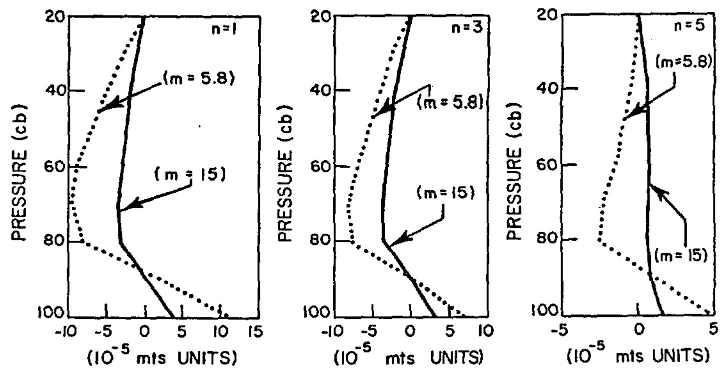

FIGURE 9.—Same as figure 8 for the generation term.

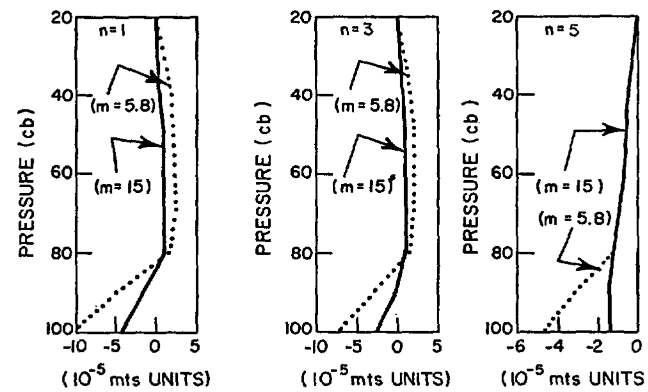

FIGURE 10.—Same as figure 8 for the heat transport term.

The term on the left describes the conversion of potential energy into kinetic energy [$\cos(\alpha_\omega-\alpha_p)>0$] on a constant-pressure surface. (N.B. $\sigma<0$.) The first term on the right describes the generation of potential energy on a constant-pressure surface. Warm air is being heated and cold air is being cooled when $\cos(\alpha_H-\alpha_p)<0$. The second term on the right involves the product $-k\bar{\psi}\lambda_0$, which may be rewritten $\bar{v}\lambda_0$, where $-k\bar{\psi}=\bar{v}$ and $\bar{v}$ is the amplitude of the south-north component of the wind perturbation. The term describes the change in potential energy due to the horizontal transport of heat and treats the conversion of mean available potential energy into eddy available potential energy. Potential energy is increased if cold air is transported southward and warm air northward [$\sin(\alpha-\alpha_p)<0$].

It has been shown that $\alpha=\pm\pi/2$ when $K_0=0$ (no friction). This condition requires that $\alpha_p=\pm\pi/2$. Therefore, $\cos(\alpha_H-\alpha_p)=0$ and $\sin(\alpha-\alpha_p)=0$, which infers that regions of generation and dissipation of potential energy both exist on a constant-pressure surface, that their net effect is to produce a zero generation term, and that there is no horizontal transport of heat. These conditions yield

$$\sigma\omega_0\lambda_0\cos(\alpha_\omega-\alpha_p)=0. \tag{18}$$

Since neither σ, ω_0, nor λ_0 are zero, then

$$(\alpha_\omega-\alpha_p)=\pm\pi/2. \tag{19}$$

Therefore, both thermally direct and thermally indirect circulations exist in the atmosphere, and the conversion of potential energy into kinetic energy by the thermally direct circulation is compensated completely by the conversion of kinetic energy into potential energy by the thermally indirect circulations.

When friction is introduced, the pressure wave displaces continuously and $(\alpha_H-\alpha_p)\neq\pm\pi/2$ and $(\alpha-\alpha_p)\neq\pm\pi$; therefore, the generation term, the conversion term, and the horizontal heat transport term are different from zero. Figures 8, 9, and 10 show the vertical distribution of these terms for wave numbers 1, 3, and 5 for the conditions specified earlier.

The conversion term (fig. 8) is zero at the upper and lower boundary because of the boundary conditions. For $n=1$ and 3 and for $n=5$ and $m=5.8$, the magnitude is a maximum between 70 and 80 cb, the magnitude increases as friction is allowed at lower pressures, and there is a conversion of potential energy into kinetic at all interior levels. Near the resonance state ($n=5$ and $m=15$), potential energy is converted into kinetic energy at high pressure and kinetic energy into potential energy at low pressure.

For $n=1$ and 3 and for $n=5$ and $m=5.8$, there is a degeneration of potential energy (positive values) at high pressure (fig. 9), and a generation of potential energy at low pressure. However, pressure integration of the generation term yielded a net generation of potential energy. The magnitude of this term also increased as friction was allowed at lower pressure. Near the resonance mode ($n=5$ and $m=15$), all levels indicate a degeneration of potential energy.

For $n=1$ and 3, northward transport of heat was found at high pressure (fig. 10) with negligible southward transport of heat at low pressure. However, there was northward transport of heat at all levels for $n=5$, which corresponds to the westward tilt of the pressure systems. This term also increased notably as friction was allowed at lower pressure.

The following expression for the change in kinetic energy on a constant-pressure surface was derived using eq (3), following a technique similar to that used to derive eq (19):

$$k^2K\bar{\psi}^2 = f\frac{\partial}{\partial p}[\bar{\psi}\omega_0 \cos(\alpha_\omega - \alpha)] + f\omega_0\lambda_0 \cos(\alpha_\omega - \alpha_p). \quad (20)$$

The loss of kinetic energy due to friction is compensated by the vertical compression or expansion of the system and by the conversion of potential energy into kinetic energy. The interpretation of eq (20) for this particular case can be readily seen if the pressure integral of eq (20) is obtained. Integrating and applying the boundary conditions yields

$$\frac{1}{\delta p}\int_{p_s}^{p_0} k^2K\bar{\psi}^2 dp = \int_{p_s}^{p_0} \omega_0\lambda_0 \cos(\alpha_\omega - \alpha_p)dp. \quad (21)$$

Equation (21) indicates that the frictional loss is balanced by the conversion of potential energy into kinetic energy only. Therefore, as friction is allowed at lower pressure, there must be a corresponding increase of the conversion of potential energy into kinetic energy as is indicated in figure 8.

5. CONCLUSION

The resonance mode, characteristic of a frictionless atmosphere, was apparent when friction was concentrated in the boundary layer. When relatively significant friction was allowed at low pressure, the influence of the resonance was not found. This suggests that the response of the atmosphere to surface heating or to any other forcing function should be considerably different over the oceans where the roughness lengths are small and friction is confined to the low levels, than over the continents where the roughness lengths can be large and friction can be considerable at high levels. Furthermore, frictional effects should be much more apparent at lower pressures in the summer months than in the winter months because of convection. Therefore, the resonance mode should be more discernable in winter. Data presented by Saltzman and Fleisher (1962) seem to indicate this effect.

Friction generally reduced the amplitude of the steady-state disturbance due to surface heating at low levels and increased the disturbance at high levels. This circumstance was due to the frictional effect acting in a sense opposite to the divergence at low levels, requiring a small perturbation to maintain a steady state, and acting in the same sense as the divergence at upper levels requiring a larger perturbation to maintain a steady state. However, near the resonance wave number, the above characteristics did not hold.

The pressure systems were displaced with decreasing pressure in response to the change in sign of the divergence. The displacement was continuous when friction was included. The pressure systems were found to tilt eastward when the pressure-averaged Rossby speed was indicating westward motion ($C_R<0$) and westward when it indicated eastward motion ($C_R>0$). This is a result of the change in sign of the dynamic transport of momentum. The slope of the pressure systems decreased when significant friction was allowed at lower pressures.

In a steady state with surface heating, the conversion of potential energy into kinetic energy for the long waves was balanced by the generation of potential energy and by the northward transport of heat. However, when friction was zero, the net generation on an isobaric surface was zero, indicating that local zones of generation were exactly canceled by local zones of degeneration, and the heat transport was zero. The conversion term was also zero, which implies the positive energy conversions associated with thermally direct circulations are exactly balanced by the negative conversions associated with thermally indirect circulations. When friction was introduced, there was a conversion of potential energy into kinetic energy, a degeneration of potential energy at high pressures and a generation at low pressures, and a northward transport of heat at high pressures with negligible southward transport of heat at low pressures for all the long waves except near the resonance mode. At low pressure, the conversion term was, to a great extent, balanced by the generation term. The magnitude of each term increased as friction was increased at low pressure.

On the average, these results suggest that in a steady state the eddy conversion term is balanced by the eddy generation term, and the conversion of mean available potential energy into eddy available potential energy is negligible. This is contrary to observed results (Brown 1964, Wiin-Nielsen 1964, and Dutton and Johnson 1967), which suggest that on the average, there is a degeneration of eddy available potential energy, and the major source of eddy available potential energy is through the conversion of mean available potential energy into eddy available potential energy. Differences may be attributed to the fact that the observed results treat these terms in a nonsteady state and are summations over all wave numbers, including the effect of a y variability.

In terms of kinetic energy, the total frictional loss was balanced by the conversion process, which explains why the conversion term increased as friction was allowed at lower pressure. The generation term and the northward heat transport term responded to the increase in the conversion term.

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